Schmidt Norms for Quantum States

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2. Schmidt Vector Norms
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Motivation

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- The Schmidt vector norms were used by Chruscinski and Kossakowski in early 2009 to develop a test for k-positivity of linear maps.
- A conjecture of Brandao (which would imply $QMA(k) = QMA(2)$ for $k > 2$) involves the Schmidt operator 1-norm.

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But first...

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- $SR(|v\rangle)$ is the Schmidt rank (a.k.a. the tensor rank) of the bipartite pure state $|v\rangle$. $SN(\rho)$ is the Schmidt number of the bipartite density operator ρ.

k-Block Positive Operators

An operator $X = X^* \in \mathcal{L}(\mathcal{H}_n) \otimes \mathcal{L}(\mathcal{H}_n)$ is said to be **k-block positive** if

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- The n-block positive operators are exactly the positive operators, since $SR(|v\rangle) \leq n$ for all vectors $|v\rangle$.
- These operators are sometimes referred to as ***k*-entanglement witnesses** because $SN(\rho) \leq k$ if and only if

$$\mathrm{Tr}(X\rho) \geq 0 \quad \forall\, k\text{-block positive } X.$$

Schmidt Vector Norms

Let $|v\rangle \in \mathcal{H}_n \otimes \mathcal{H}_n$ and let $1 \leq k \leq n$. Then we define the **Schmidt vector *k*-norm** of $|v\rangle$ by

$$\big\||v\rangle\big\|_{s(k)} := \sup_{|w\rangle}\Big\{\big|\langle w|v\rangle\big| : SR(|w\rangle) \leq k\Big\}.$$

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Computation

From now on, when we refer to the Schmidt coefficients $\{\alpha_i\}$ of a vector, we will assume that they are written in non-increasing order (i.e., $\alpha_1 \geq \alpha_2 \geq \cdots \geq \alpha_n \geq 0$).

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The Schmidt vector norms are actually very simple to compute, as the following theorem demonstrates:

Theorem

Let $|v\rangle \in \mathcal{H}_n \otimes \mathcal{H}_n$ have Schmidt coefficients $\{\alpha_i\}$. Then

$$\big\| |v\rangle \big\|_{s(k)} = \sqrt{\sum_{i=1}^{k} \alpha_i^2}.$$

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- Computing Schmidt operator norms (even just for positive operators) is equivalent to the problem of determining k-block positivity of an operator.
- Determining k-block positivity has been extensively studied by mathematicians for some 30 years, but no complete method is known. This suggests that computing Schmidt operator norms is very difficult.

Computation

Nonetheless, we can derive many inequalities to bound the Schmidt operator norms (or even compute them exactly) in certain situations. These results lead to a multitude of tests for k-block positivity, which help us tackle the NPPT bound entanglement problem.

Testing k-Block Positivity

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- We derive a general result for testing k-block positivity based on Schmidt operator norms. We then use our inequalities to derive "computable" tests for k-block positivity.
- Known tests that follow from our general result include Takesaki and Tomiyama (1983), Benatti, Floreanini, and Piani (2004), Kuah and Sudarshan (2005), and Chruscinski and Kossakowski (2009).

Testing k-Block Positivity

Because the general result is quite technical, we present only some of the *new* tests that follow from it.

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Theorem (number of negative eigenvalues)

Suppose $X = X^* \in \mathcal{L}(\mathcal{H}_n) \otimes \mathcal{L}(\mathcal{H}_n)$ is k-block positive. Then it has at most $(n-k)^2$ negative eigenvalues.

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Theorem (magnitude of negative eigenvalues)

Suppose $X = X^* \in \mathcal{L}(\mathcal{H}_n) \otimes \mathcal{L}(\mathcal{H}_n)$ is k-block positive. Denote the maximal and minimal eigenvalues of X by λ_{max} and λ_{min}, respectively. Then

$$\frac{\lambda_{min}}{\lambda_{max}} \geq 1 - \frac{n}{k}.$$

Testing k-Block Positivity

The following result “interpolates” between the two results on the previous slide.

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Theorem (number and magnitude of negative eigenvalues)

Suppose $X = X^* \in \mathcal{L}(\mathcal{H}_n) \otimes \mathcal{L}(\mathcal{H}_n)$ is k-block positive with r negative eigenvalues. Denote the maximal and minimal eigenvalues of X by λ_{max} and λ_{min}, respectively. Then

$$\frac{\lambda_{min}}{\lambda_{max}} \geq 1 - \frac{\left\lceil \left(n - \sqrt{r-1}\right) \right\rceil}{k} \quad \textit{and}$$

$$\frac{\lambda_{min}}{\lambda_{max}} \geq 1 - \frac{n^2(n-1)}{(k-1)n^2 + (n-k)r}.$$

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Theorem

Let $X = X^* \in \mathcal{L}(\mathcal{H}_n) \otimes \mathcal{L}(\mathcal{H}_n)$ have exactly two distinct eigenvalues $\lambda_1 > \lambda_2$. Let P^- be the projection onto the negative part of X. Then X is k-block positive if and only if

$$\|P^-\|_{S(k)} \leq \frac{\lambda_1}{\lambda_1 - \lambda_2}.$$

Testing k-Block Positivity

The result on the previous slide is useful for helping tackle the problem of whether or not there exist NPPT bound entangled states. The rest of this talk will concern this problem, so we will now introduce Werner states and bound entanglement.

Bound Entangled States

A state $\rho \in \mathcal{L}(\mathcal{H}_n) \otimes \mathcal{L}(\mathcal{H}_n)$ is said to be **bound entangled** if $(id_n \otimes T)(\rho)^{\otimes \ell} \in \mathcal{L}(\mathcal{H}_n^{\otimes \ell}) \otimes \mathcal{L}(\mathcal{H}_n^{\otimes \ell})$ is 2-block positive for all $\ell \geq 1$, where T denotes the transpose operation.

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- Equivalently, a state is bound entangled if it has zero **distillable entanglement**.
- This means that bound entangled states are states that can not be transformed via local operations and classical communication into a pure maximally entangled state.
- Separable states are clearly bound entangled.

Bound Entangled States

- **Positive partial transpose (PPT)** states are also bound entangled. That is, if $\rho \geq 0$ and $(id_n \otimes T)(\rho) \geq 0$ then ρ is bound entangled.

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- It has been an open question for over a decade whether or not other bound entangled states exist.
- That is, does there exist a bound entangled state ρ with non-positive partial transpose (NPPT)?

Bound Entangled States

- Mathematically, does there exist a density operator ρ such that $(id_n \otimes T)(\rho) \ngeq 0$ yet $(id_n \otimes T)(\rho)^{\otimes \ell}$ is 2-block positive for all $\ell \geq 1$?

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- It has been shown that it is enough to consider Werner states – that is, there exist NPPT bound entangled states if and only if there exist NPPT bound entangled Werner states.

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- Mathematically, does there exist a density operator ρ such that $(id_n \otimes T)(\rho) \not\geq 0$ yet $(id_n \otimes T)(\rho)^{\otimes \ell}$ is 2-block positive for all $\ell \geq 1$?
- It has been shown that it is enough to consider Werner states – that is, there exist NPPT bound entangled states if and only if there exist NPPT bound entangled Werner states.
- So let's look at Werner states!

Werner States

Let $S \in \mathcal{L}(\mathcal{H}_n) \otimes \mathcal{L}(\mathcal{H}_n)$ be the swap operator that maps $|a\rangle \otimes |b\rangle$ to $|b\rangle \otimes |a\rangle$. Then **Werner states** are the density operators of the following form, which are parametrized by a single real variable $\alpha \in [-1, 1]$:

$$\rho_\alpha := \frac{1}{n^2 - \alpha n}(I - \alpha S) \in \mathcal{L}(\mathcal{H}_n) \otimes \mathcal{L}(\mathcal{H}_n).$$

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- From now on we will ignore the scaling factor of $\frac{1}{n^2-\alpha n}$ in front of the Werner state, since positive scalars don't affect block positivity.

Bound Entangled Werner States

The partial transpose of a Werner state is of the form

$$(id_n \otimes T)(\rho_\alpha) = I - \alpha n E,$$

where $E := \frac{1}{n}\sum_{i,j=1}^{n} |i\rangle\langle j| \otimes |i\rangle\langle j|$ is the projection onto the "standard" pure maximally entangled state.

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- This operator has only 2 distinct eigenvalues! This means that our k-block positivity test from earlier applies in this situation.

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Theorem

Let ρ_α be a Werner state. Then $(id_n \otimes T)(\rho_\alpha)$ is k-block positive if and only if $\alpha \leq \frac{1}{k}$.

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- Two special cases of this result are well-known. First, ρ_α has positive partial transpose if and only if $\alpha \leq \frac{1}{n}$. Second, ρ_α is 2-block positive if and only if $\alpha \leq \frac{1}{2}$.

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- This shows that the "interesting region" of α's is $\alpha \in (\frac{1}{n}, \frac{1}{2}]$, since ρ_α is PPT for $\alpha \leq \frac{1}{n}$ and $(id_n \otimes T)(\rho_\alpha)$ is not 2-block positive (and hence ρ_α is not bound entangled) for $\alpha > \frac{1}{2}$.

Bound Entangled Werner States

Numerical evidence suggests that ρ_α is bound entangled for all $\alpha \in (\frac{1}{n}, \frac{1}{2}]$. Actually proving this seems to be out of reach, although we are able to rephrase the problem in terms of Schmidt operator norms...

Bound Entangled Werner States

Theorem

For $n \geq 4$, the state $\rho_{2/n}$ is bound entangled if and only if $\lim_{\ell\to\infty} \big\|P_\ell\big\|_{S(2)} = \frac{1}{2}$, where P_ℓ is the orthogonal projection defined recursively via

$$P_1 := E,$$
$$P_{\ell+1} := E \otimes (I - P_\ell) + (I - E) \otimes P_\ell, \ \text{for } \ell \geq 1.$$

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So get calculating!

Concluding Remarks

- Schmidt norms are powerful tools for determining k-block positivity of Hermitian operators.

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- A method of computing these norms on a certain "highly entangled" family of projections would help solve the NPPT bound entanglement problem.

Further Reading

N. Johnston, D. W. Kribs, *Schmidt Norms for Quantum States*, preprint (2009). arXiv:0909.3907 [quant-ph]